2.27 ANGLE TRACKING

Tracking radars are closed loop systems that attempt to keep the selected target centered within the beam scan pattern and provide tracking data to a fire control system. The primary output of most radar tracking systems is the target location determined by the pointing angles of the antenna beam and the positions of the range tracking gates. The tracking data is used by a fire control computer to predict the future position of the target in order to achieve an intercept. For anti-aircraft artillery, the prediction is for just a few seconds into the future, the flight time of the projectiles. Angle tracking systems extract and process target off-boresight error measurements to reposition antenna pointing servos in an attempt to maintain small error measurements. This functional element is concerned with the angle tracking circuitry that keeps the antenna pointed at the target and provides target position and rate data to the fire control computer.

Two major components are imbedded in the angle track loop illustrated in Figure 2.27-1: the angle discriminator and the element which repositions the antenna boresight. The angle discriminator determines the angle error between the target and boresight. This is used to drive the antenna servos.

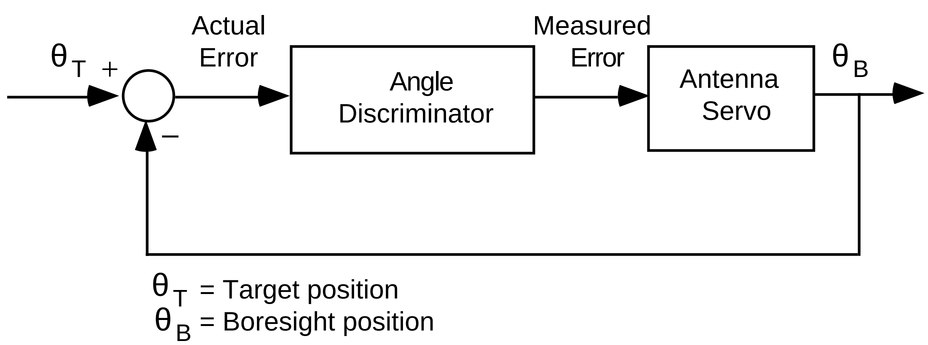

FIGURE 2.27-1. Angle Track Loop.

One type of angle tracker used by AAA radars is the conical scan (CONSCAN) radar. During autotrack, the transmitted beam of a CONSCAN radar nutates in a conical scan pattern about the antenna boresight. This motion periodically points the beam closer to and farther from the target, causing the target return to fluctuate sinusoidally during a scan. The amplitude of this signal decreases as the target approaches the antenna boresight, and a target on boresight produces an amplitude of zero; i.e., no error signal is generated because the target return is constant. The magnitude and phase of the sinusoidal signal determine the magnitude and direction of the azimuth and elevation tracking error signals.

2.27.1 Functional Element Design Requirements

This section discusses the design requirements for the angle tracking functional element (angle servo only).

a. The angle tracking functional element will simulate the motion of the antenna in response to the estimated angle errors from the angle discriminators. Once the boresight errors are estimated, the antenna is moved to null the error using two servos, one for azimuth and one for elevation. The antenna boresight position will be updated at the conical scan rate, subject to gimbal and rate limits.

b. The angle tracking functional element will provide for the capability to bypass radar angle tracking if the radar has lost the target. Radar loss of a target may occur due to target masking by a user-defined hill, angle errors caused by excessive angular tracking rate, or a decrease in target signal return.

2.27.2 Functional Element Design Approach

Equations [2.27-1] through [2.27-6] provide the background information for the discrete solution for the antenna servo response described in Design Element 27-1.

Antenna Servo Response (Frequency Domain). The antenna servos will be modeled as closed-loop controllers described in the frequency domain by a transfer function of the following form:

$$H(s) = \frac{K_4 s + K_3}{s^3 + K_2 s^2 + K_1 s} \qquad [2.27\text{-}1]$$

where the K_i are system parameters, generally different for azimuth and elevation, and varying from radar to radar.

Antenna Servo Response (Time Domain). The response of the servo system to an input (excitation) function, u(t), is given by the inverse Laplace transform of Equation [2.27-1]:

$$\dddot{x}(t) + K_2 \ddot{x}(t) + K_1 \dot{x}(t) = K_3 u(t) + K_4 \dot{u}(t) \qquad [2.27\text{-}2]$$

where *x(t)* = new boresight angle (azimuth or elevation)

u(t) = angle error passed from discriminator (azimuth or elevation)

dots represent differentiation with respect to time (*t*)

Antenna Servo Response (General Solution). Equation [2.27-2] can be reduced to a linear system of coupled, first order differential equations by the following substitution:

$$x_1 = x\,,\quad x_2 = \dot{x}\,,\quad x_3 = \ddot{x}\,,\quad x_4 = \dddot{x} \qquad [2.27\text{-}3]$$

Equations [2.27-2] and [2.27-3] can be expressed in matrix form as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -K_1 & -K_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ K_3 & K_4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ \dot{u} \\ \ddot{u} \\ \dddot{u} \end{bmatrix} \qquad [2.27\text{-}4]$$

Equation [2.27-4] is in the form of the classical state-space equation

$$\left[\dot{x}(t)\right] = [A]\left[x(t)\right] + [B]\left[u(t)\right] \qquad [2.27\text{-}5]$$

where: $[\dot{x}]$ = matrix of derivatives of the state variables
[A] = matrix of constants determined by the system parameters
[x] = matrix of state variables
[B] = matrix of constants describing the inputs
[u] = matrix of system forcing (excitation) functions

The general solution to Equation [2.27-5] is composed of a complementary solution (no forcing function) and a particular solution:

$$\left[x(t)\right] = e^{[A]t}\ \left[x(0)\right] + e^{[A]t}\int_0^t e^{-[A]r}[B]\ \left[u(r)\right]dr \qquad [2.27\text{-}6]$$

Design Element 27-1: Antenna Servo Response (Discrete Solution)

Since the angle discriminator provides updates only once every scan (i.e., at times $t = nT$, where T is the scan time), a discrete version of Equation [2.27-6] is required. This is obtained by approximating $u(t)$ over each interval $[kT, (k+1)T]$ by $u(kT)$. Thus for $t = nT$ (n = 0,1,2,...), the integral in Equation [2.27-6] can be rewritten as a sum of integrals with $u(t)$ constant in each:

$$\left[x(nT)\right] = e^{[A]nT}\left[x(0)\right] + e^{[A]nT}[B]\sum_{i=0}^{n-1}u(kT)\int_{kT}^{(k+1)T} e^{-[A]r}dr \qquad [2.27\text{-}7]$$

For ease of calculation $[x(nT)]$ can be defined recursively as follows:

$$\left[x((k+1)T)\right] = \left[\Phi(T)\right]\left[x(kT)\right] + \left[\quad (T)\right]\left[u(kT)\right] \qquad [2.27\text{-}8]$$

where:

$$\left[\Phi(T)\right] = e^{[A]T} \qquad [2.27\text{-}9]$$

and

$$\left[\quad (T)\right] = e^{[A]T}\ [B]\int_0^T e^{-[A]r}dr \qquad [2.27\text{-}10]$$

Note that $\Phi(T)$ and $\quad(T)$ depend only on the scan time T, not on k, and thus need to be calculated only once.

Equation [2.27-8] follows from substituting [2.27-9] and [2.27-10] into [2.27-7] evaluated at $n = k+1$, and by noting that

$$e^{-[A]kT}\int_0^T e^{-[A]r}dr = \int_{kT}^{(k+1)T} e^{-[A]s}ds \qquad [2.27\text{-}11]$$

by the substitution $s = r + kT$.

Design Element 27-2: Taylor Series Expansions

The functions Φ and will be evaluated by means of Taylor series expansions; these are expressed as:

$$[\Phi(T)] = [I] + [A]T + [A]^2\frac{T^2}{2} + \qquad [2.27\text{-}12]$$

and

$$[\;\;(T)] = [B]\left\{[I]T - [A]\frac{T^2}{2} + [A]^2\frac{T^3}{3!} - [A]^3\frac{T^4}{4!} + \right\} \qquad [2.27\text{-}13]$$

where $[I]$ is a 4x4 identity matrix and all other terms have been previously defined.

These series will be evaluated through the T^{10} terms. This will allow approximately 11 decimal places of accuracy.

Design Element 27-3: Clamping of Error Signals

In the antenna servo, limits are applied to the input signals to account for the motor/clutch response as follows:

$$\begin{aligned} &IF\ |\theta| < 0.10, \quad THEN\ \theta = 0.0001 \cdot sign(\theta) \\ &IF\ |\theta| > 2.0, \quad THEN\ \theta = 2.0 \cdot sign(\theta) \end{aligned} \qquad [2.27\text{-}14]$$

where θ can represent either the azimuth error or the elevation error voltage passed from the discriminator.

Design Element 27-4: Scaling Factors

The input angle error voltages (denoted by θ_{az} and θ_{e1} are scaled for a fast and stable response. The azimuth and elevation scaling factors, K_{az} and K_{el}, were empirically derived through software testing and are system dependent. These factors are multiplied times the input angle error voltages to produce clamped error voltages (θ'):

$$\begin{aligned} \theta'_{az} &= K_{az}\theta_{az} \\ \theta'_{el} &= K_{el}\theta_{el} \end{aligned} \qquad [2.27\text{-}15]$$

At each time t, each of these angle errors, θ', is used as the function $u(t)$ in Equation [2.27-2]. The derivatives of $u(t)$ are also necessary to find the matrix $[u(t)]$.

Design Element 27-5: Derivatives

Derivatives of *u(t)* and *x(t)* are calculated numerically:

$$\frac{d^n y(t)}{dt^n} = \frac{\dfrac{d^{n-1}y(t)}{dt^{n-1}} - \dfrac{d^{n-1}y(t-T)}{dt^{n-1}}}{T} \qquad [2.27\text{-}16]$$

for n = 1 to 4. The scan time T is the step size in these calculations.

Design Element 27-6: Slew Rate Limits

For both azimuth and elevation, the servo response x(t) is bounded by slew rate limits:

$$\begin{aligned} &If \;\; |\dot{x}_{az}(t)| > S_{az} \;\; then \; \dot{x}_{az}(t) = S_{az} \cdot sign(\dot{x}_{az}(t)) \\ &If \;\; |\dot{x}_{el}(t)| > S_{el} \;\; then \; \dot{x}_{el}(t) = S_{el} \cdot sign(\dot{x}_{el}(t)) \end{aligned} \qquad [2.27\text{-}17]$$

where: S_{az} = maximum azimuth slew rate
S_{el} = maximum elevation slew rate

The antenna may rotate 360 deg in azimuth, but is limited in elevation:

$$\begin{aligned} &If \;\; x_{el}(t) > M_{el} \;\; then \;\; x_{el}(t) = M_{el} \\ &If \;\; x_{el}(t) < m_{el} \;\; then \;\; x_{el}(t) = m_{el} \end{aligned} \qquad [2.27\text{-}18]$$

where: m_{el} = minimum antenna elevation
M_{el} = maximum antenna elevation

Design Element 27-7: Manual and Memory Modes

To implement the bypass requirement, the response of the servos to the radar angle errors is bypassed if manual or memory mode is in use.

If the system is in memory mode, the target signal has been lost due to masking behind a user-defined hill. If this has occurred, the FCC has predicted linear target movement based on the last known target direction and velocity. The servo is directed to set the azimuth and elevation of the antenna to these values:

$$\begin{aligned} &x_{az}(t) = \theta_{az} \\ &x_{el}(t) = \theta_{el} \end{aligned} \qquad [2.27\text{-}19]$$

where: θ_{az} = azimuth angle passed from FCC
θ_{el} = elevation angle passed from FCC

If the system is in manual mode, it is simulating a manual reacquisition mode after a breaklock on an unmasked target. The operator is assumed to move the antenna linearly back onto the target; thus, the azimuth and elevation servo responses are set to the values computed by the operator.

$$x_{az}(t) = \theta_{az}$$
$$x_{el}(t) = \theta_{el} \qquad [2.27\text{-}20]$$

where: θ_{az} = azimuth angle passed from operator
θ_{el} = elevation angle passed from operator

Design Element 27-8: Track Restoration Logic

To be determined.

2.27.3 Functional Element Software Design

This section contains the software design necessary to implement the functional element requirements and design approach described above. The first subsection contains the subroutine hierarchy and descriptions. The second subsection presents the logical flow chart and describes each block therein. The last subsection lists and describes the input and output variables.

Angle Tracking Subroutine Design

RADGUNS employs subroutine MOVANT to simulate the antenna servo system, moving the antenna in response to error signals from the receiver. The radar receiver model computes two tracking angle error signals on a scan-by-scan basis and sends them to MOVANT to calculate the response of the antenna servos. The angle error signals and the current antenna boresight angles are input to the angle servo models which return a new value for the antenna boresight angles.

Figure 2.27-2 shows the call tree for the angle servo functional design in the *RADGUNS* source code. RCVRT, the top routine in the tracking radar implementation, calls MOVANT to implement both elevation and azimuth angle error handling.

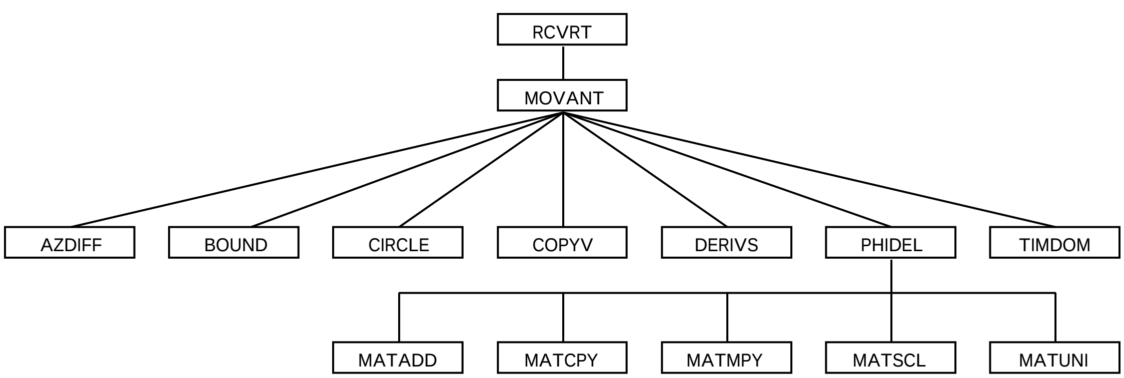

FIGURE 2.27-2. Angle Servo Subroutine Call Tree.

All of the subroutines called by MOVANT are mathematical utilities. Table 2.27-1 contains a brief description of each of these subroutines.

TABLE 2.27-1. Math Utility Routines called by MOVANT.

Name	**Description**
AZDIFF	Returns difference between two azimuth angles
BOUND	Returns a value for a variable that is within upper and lower bounds
CIRCLE	Converts any angle to an equivalent angle between 0 and 2π
COPYV	Copies a vector
DERIVS	Calculates derivatives using differences between successive values
PHIDEL	Computes the Φ and matrices for the time domain solution of a differential equation
TIMDOM	Calculates the time domain solution of a differential equation
MATADD	Matrix addition
MATCPY	Copies one matrix to another
MATMPY	Matrix multiplication
MATSCL	Multiplies a matrix by a scalar
MATUNI	Initializes a matrix to the identity matrix

Logical Flow Diagram for Angle Tracking. The logical flow of subroutine MOVANT is shown in Figure 2.27-3. The figure contains input, output and internal variables that will be discussed in the next section. The blocks are numbered for ease of reference in the following discussion.

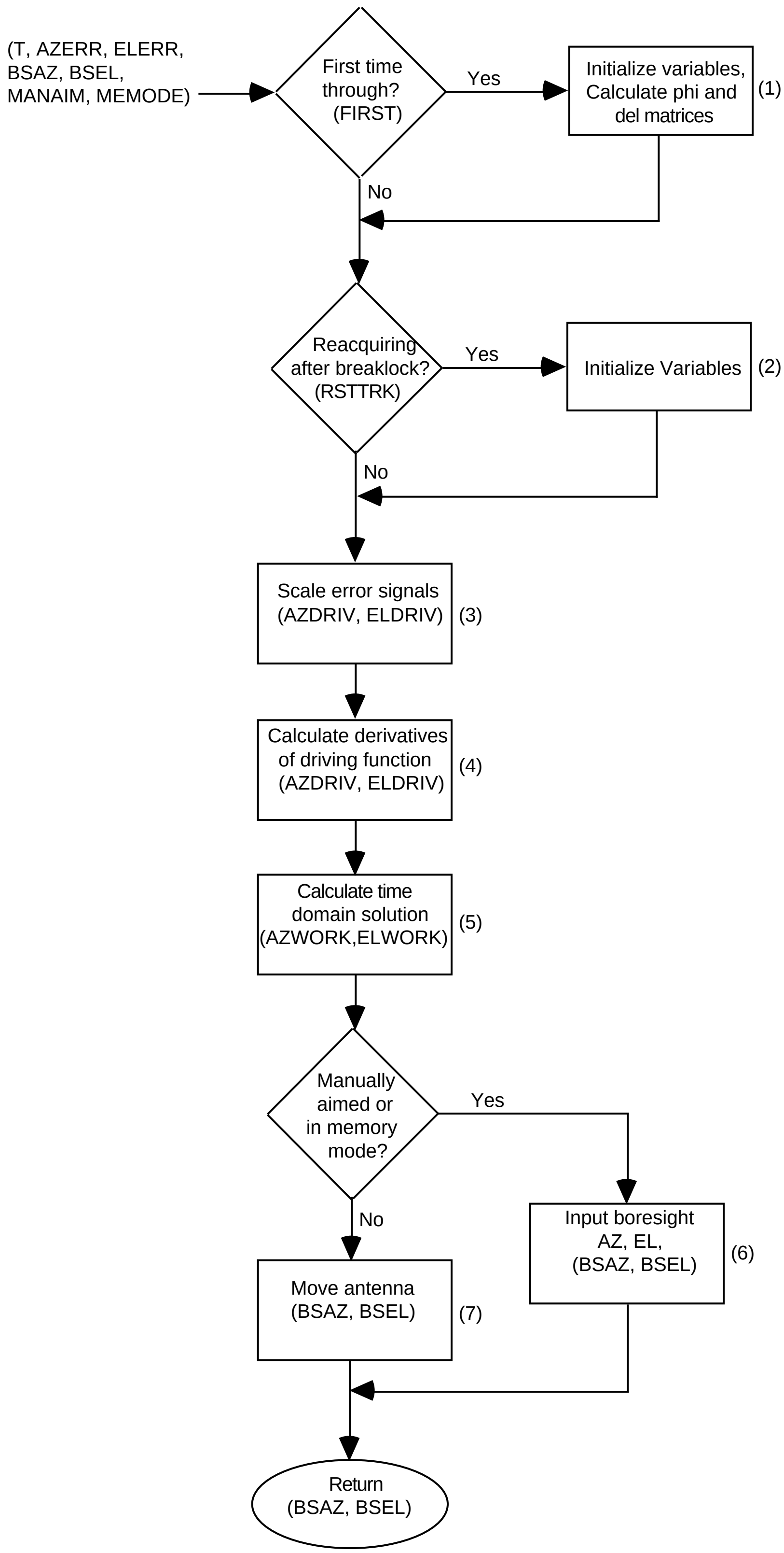

FIGURE 2.27-3. MOVANT Subroutine Flow Chart.

Block 1. On the first call to MOVANT, several variables are initialized. The antenna driving functions (*u(t)* of Equation [2.27-2] for both azimuth and elevation) and their derivatives are initialized to zero. If the system is manually aimed or in memory mode, the previous boresight values are also set to zero. The servo outputs (*x(t)* in Equation [2.27-2] for both azimuth and elevation) are initialized to the current boresight values. The step size is set to the scan time (T), and the matrices [A] and [B] of Equation [2.27-5] are initialized with the correct system values for both azimuth and elevation. Subroutine PHIDEL is called to calculate $\Phi(T)$ and (T) using Equations [2.27-12] and [2.27-13]. Subroutine PHIDEL calls the matrix utilities shown in Figure 2-25 to implement these matrix equations.

Block 2. If the target has been reacquired after a break lock, some variables need reinitializing. All variables are set to the same values described in Block 1. The values of T, $\Phi(T)$, and (T) have not changed and need not be recalculated.

Block 3. The azimuth and elevation angle errors are clamped and scaled as described in Equations [2.27-14] and [2.27-15].

Block 4. The derivatives of the angle errors are calculated by a call to DERIVS, which performs the derivation shown in Equation [2.27-16].

Block 5. The responses of the azimuth and elevation servos to the errors passed from the discriminator are then calculated. Subroutine TIMDOM is called to calculate [*x(t)*] in Equation [2.27-2], using Equation [2.27-8] unless this is the first time through. TIMDOM, in turn, calls MATMPY and MATADD for matrix multiplication and addition, respectively. Function BOUND is used to ensure gimbal limits and slew rate limits are not exceeded, as described in Equations [2.27-17] and [2.27-18].

Block 6. At this point, a check is made to determine if, in fact, the radar system is still tracking the target. If the radar is not tracking the target, it is in memory or manual mode, as described in Section 7.1.3.10. In these cases, the angles predicted by the FCC or the operator are passed to subroutine MOVANT and used as the values of antenna boresight azimuth and elevation. Finally, the boresight azimuth and elevation angles are stored for the next call to MOVANT.

Block 7. If the system is not in memory or manual mode, the boresight angles are updated to the values calculated in Block 5. This simulates moving the antenna. These values are stored for the next call to MOVANT.

Functional Element Inputs and Outputs

The user may select one of five track modes shown in Table 2.27-2. Only two of these modes, RADR and OPCH, rely on radar tracking. In RADR track mode, the user may choose the L-option to lock one or more dimensions (azimuth, elevation, or range) of the radar on the true target position. If all dimensions are locked on the target, the radar is bypassed and the antenna boresight is set to the true target position. If radar tracking is selected in any dimension, the respective tracking error is calculated as described above and passed to the applicable servo. In optical cross-hairs tracking, the radar provides range information only.

TABLE 2.27-2. User Input Data.

Track Mode	Description	Subroutines
BARF	Barrage fire in one direction only (no tracking)	BARFIR
OPCH option	Optical cross-hairs tracking in AZ and EL Radar range tracking with N, L, or A option	OPTRAK, RCVRT, RSERVO
OPCS	Optical cross-hairs tracking in AZ and EL RG mechanically input by operator	OPTMCS OPTRAK
OPSR	Optical tracking in AZ, EL with speed rings prediction	SPDRNG, OPTRAK
RADR option	Radar tracking with three options N: full radar tracking in AZ, EL, and RG L: one or more dimensions locked on target A: operator range assist (halves radar range tracking errors)	RCVRT, MOVANT, RSERVO depending on dimensions locked

In addition to the user inputs listed in Table 2.27-2, there are numerous constants which the user effectively inputs by selecting a specific radar system. These are listed in Table 2.27-3.

TABLE 2.27-3. Radar-Specific Input Variables for Angle Servos.

Math Symbol	Variable	Description
K_{az}	KAZ	Azimuth Scale Factor (empirical)
K_{el}	KEL	Elevation Scale Factor (empirical)
K_i	Entries in matrices [A] & [B]	Filter constants, AZ and EL channels.
M_{el}	AMAXEL	Maximum Elevation (Gimbal) Limit (rad)
m_{el}	AMINEL	Minimum Elevation (Gimbal) Limit (rad)
S_{az}	AMXAZV	Maximum Azimuth Slew Rate (rad/s)
S_{el}	AMXELV	Maximum Azimuth Slew Rate (rad/s)
T	SCANT	Conical Scan Period (s)

Table 2.27-4 contains a description of the input variables for MOVANT. BSAZ and BSEL, the boresight azimuth and elevation position, are both inputs and outputs for MOVANT.

TABLE 2.27-4. Subroutine MOVANT Input/Output Variables.

Variable Name	Input/Output Type	Output By	Description
AZERR	Argument	RCVRT	Azimuth error signal from phase detector
BSAZ	Argument	REACQR MOVANT	Antenna boresight azimuth
BSEL	Argument	REACQR MOVANT	Antenna boresight elevation
ELERR	Argument	RCVRT	Elevation error signal from phase detector
MANAIM	Argument	ENGAGE	True if manual slewing
MEMODE	Argument	ENGAGE	True if in memory mode
T	Argument	ENGAGE	Time in scenario

2.27.4 Assumptions and Limitations

The Taylor Series expansion is an approximation to the actual time domain solution. However, the expansion to 10 terms provides accuracy approaching that of most floating point arithmetic processors.

No angle error sources are considered other than those generated by the discriminator. This is not considered a major problem, since servo noise is typically very small compared to sources such as receiver noise, glint and scintillation, and multipath.

It is assumed that no errors are introduced to the prediction from the FCC or the operator when the radar is in manual or memory mode.